Providing a framework to assess the competitive location of bank branches using game theory

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Abstract

In recent years, growth of financial and credit institutions such as banks, the competition among them and customer satisfaction has made it essential to use modern scientific methods to choose the optimum location for them. Making optimum decision in choosing the location of the financial institution reduces costs, increases efficiency and competitiveness with other companies. In some real-world problems, the decision to locate the facility is made by independent decision-makers that in addition to maximizing the benefits, compete to maintain or increase their market share. Therefore, a new branch in location has addressed the impact of competitive conditions in determining the optimal location of facilities.

The present study aims to select the location of bank branches according to the existing competitors in the market and organizations that are likely to enter the market based on game theory and cooperative game approach. For this purpose, a model of game theory is offered using which one can determine the location of the new facility and obtain an equilibrium based on the concept of Nash equilibrium for the entire series. After the structure of cooperative games, value of each player is calculated according to the intended strategies for each player and coalitions' utility. Finally, the decision-making process is done and the optimal point to establish bank branches are selected.

Keywords: Bank branches, competitive location, game theory, cooperative game, Nash equilibrium, Shapley value

Received 23/04/2018
Approved 10/06/2018

1. Introduction

Selecting the optimum location of facilities is one of the most important and most strategic decisions in various industries that has a significant effect on the performance of organizations (Ahmadi et al, 2010). Ability of managers to create a comprehensive compilation of all aspects of the location provides favorable conditions for survival and success of the organization in the competitive environment of the industry. Creating new facilities requires a detailed study on the way of correct establishment of these facilities in different regions of the city, so that the first key point is selecting the optimum location according to different and sometimes contradictory conditions (Mousavi et al, 2013). Spending exorbitant costs in order to create business units and also paying attention to communication and ease of access indicate the importance of finding the accurate and suitable location for this group of economic actors so that easy and quick access to these firms is provided to most citizens. Bank, as part of the firms that are always in contact with people, have a special sensitivity in choosing the right place to maximize their market share and increase customer satisfaction through fast access. Also taking into account the competitive environment of banks to attract potential customers, optimal positioning of banking facilities is particularly important for managers (Alhaffa et al., 2011). The optimal policy of location is considered for three and more facilities in a linear market along which demand is distributed uniformly. In order to solve this problem, pure Nash and Stackelberg equilibrium strategy is used for games with three players in a three-stage form. In the first stage, player A chooses the optimal location of its facility based on players and strategies B and C and in the second stage, player B chooses its location based on player C. In fact, the third stage must be solved to solve the problem and solving the second stage, the first stage is solved (Shiode et al., 2013). A location game on a network where facilities are undesirable and all agencies try to be far from other facilities as much as possible has been developed. The method proposed in this study is such that first agencies announce their location and accordingly, choose their location mechanism on the network. Here, the aim is to maximize social welfare, for example, minimizing the distance between the agency and facilities. In the present study, market space is investigated as a network in different states of path, circle and tree (Cheng et al., 2014). The problem of price locating game in a discrete space is also presented, where the location of players is first determined and then, the price of their goods is determined after being placed in the locations. Profit margin is considered to be constant and demand is perfectly inelastic. In order to solve the problem correctly, branch-and-bound

Received 23/04/2018
Approved 10/06/2018

algorithm for small, medium and large sizes of heuristics is used (Fernandez et al., 2014). Given the diversity of competitive location problems with game theory approach in terms of types, structure and limits of problems and type of the objective functions, a variety of solutions have been proposed that are divided into two general categories of non-cooperative and cooperative. In most cases, the primary concepts of Nash equilibrium and Stackelberg model, Voronoi, coalition, and developmental games, rounded off equilibrium and polynomial metaheuristic algorithm are used in non-cooperative mode and Shapley, Johnston, Deegan – Packel, Shapley - Shubik and Banzhaf methods are used to solve problems in cooperative mode. However, due to the complexity of the structure of the model and solving methods of competitive location problem with cooperative game approach, number of research in this area is far less than the non-cooperative mode. Therefore, cooperative game approach using Shapley method in the field of strategic variables is used to solve competitive location problem of this study.

2. Literature Review

2.1 Basic concepts in locating

2.1.1 Facilities, location and customer are three main components describing a location allocation problem. Each of these components can be used to identify a problem and may also play different roles in different location problems (Miliotis et al., 2002).

One of the features of facilities in many location allocation models is the number of new equipment required to be placed in a specific region. Accordingly, the two general categories of single facilities and multi-facilities problems are considered. In contrast, the number of facilities required for the placement may not be known in advance. In such cases, the goal is to find the minimum number of facilities to cover all demand points in accordance with a certain distance standard (Miliotis et al., 2002). Another important feature of facilities is their type that includes capacity and services of facilities. If facilities are able to provide unlimited demands, they are called capacitated and if the capacity of facilities is limited, an incapacitated model will be faced. Cost is another feature of facilities, according to which the location allocation models can be separated. Facilities’ cost is divided into two fixed and variable types. The fixed one is dependent on expenses and the variable one associated with service provision (Miliotis et al., 2002).

2.1.2 Demand is the second essential component of location allocation algorithm that is also known as customer. Customer or demand is a person who needs to get access to a service or supply of a product. Considering the distribution of requirements in space, three different modes

Received 23/04/2018
Approved 10/06/2018

will emerge. Accordingly, requirements can be uniform in the region or on the network, can belong to a certain point (which is geographically coded) throughout the region, or can be assigned to the mass center of the region. However, when the actual data is not available, requirements can be determined randomly in order to simulate the problem. Customers' requirements can be categorized as deterministic or stochastic. If the number of customers, locations and requirements are all clear, the model is called deterministic and if the customer requirements are modeled using probability models, the model is called stochastic one (Miliotis et al., 2002).

2.1.3 The third essential component of location problems is space or location. There are three different displays of space in location problems that include discrete, continuous, and network-based display. In the discrete space model, it is assumed that there is a prior knowledge about candidate or potential locations. Hence some of the best locations will be selected from among the potential locations. Some location-allocation problems deal with continuous spaces that determine one or more coordinates with continuous changes of all potential locations. These continuous places are always considered as normal in Euclidean space (Karaganis et al., 2011).

2.2 Competitive facility location and its classification

One branch of location theory in relation to location positioning is competitive facilities. In competitive facility location models, the decision to locate the facility is made by independent decision-makers that in addition to maximizing the benefits, compete to maintain or increase their market share. Because of this competitive nature, these problems are known as competitive facility location problems (Saidani et al., 2012). Competitive facility location problems aim to find the optimal location for new facilities that must compete with existing facilities or facilities that will enter the market in the future in order to capture market share. Studies show that the input type is the basic feature to model these problems. After recognizing the problem and determining the model in terms of its input type, other parameters can be entered in the constraints or objective function (Ashtiani, 2016).

Received 23/04/2018
Approved 10/06/2018

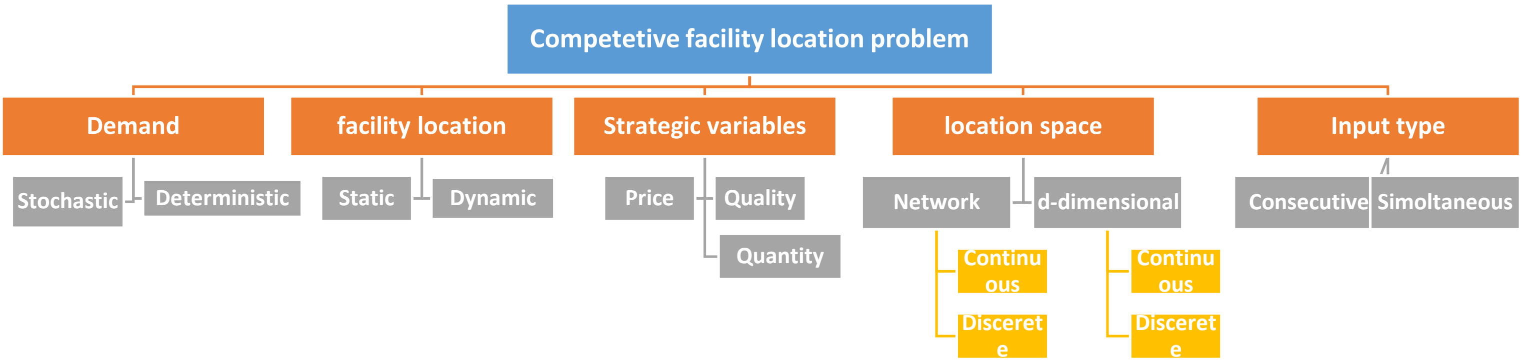

Figure 1: Classifying competitive location problems based on model's features (Ashtiani, 2016)

3. Game theory

Game theory, as a new branch of mathematics with a new approach to decision making problems, has grabbed the attention of many engineering investigators, especially industrial and civil engineering, in recent years. As the word game suggests, the problems that game theory can be applied for have several beneficiary decision makers. Game theory is a science that studies people's decisions in interactions with others. In other words, game theory is the study of conflicts and cooperation between rational actors. The main goal of game theory is providing an outlook according to which actors should behave rationally. Generally, game theory is divided into two branches of non-cooperative games and cooperative games. Non-cooperative games are those that players act independently and each player seeks to maximize his own benefits. Looking for more benefits makes them choose the Nash equilibrium that, like strategic equilibrium, is detriment of the players in most cases. In cooperative games, players may agree to choose some strategies in order to enjoy more benefits. The agreement may be between all the players or some of them, which is called coalition. The agreement must be motivated and without the use of force. In this case, players tend to cooperate when the result of cooperation is at least the same as acting independently so that the motivation to cooperate and join coalition is created in players. The first step in the cooperative game theory is searching for possible agreements or coalitions of a game and their results. The second step is dividing coalition consequences among members so that the motivation to join coalition is created. A cooperative game with n players in the form of a characteristic function is an ordered pair of G(N,v), where N is a finite set with n members, i.e. $N = \{1,2, ..., n\}$, and indeed N is the number of players. The subset (S, $S \subseteq N$) is called coalition. A coalition is easily formed when it includes the null

Received 23/04/2018
Approved 10/06/2018

set and the N itself. Also, v is a real value that shows two utility values of coalition's players and their recipient. The two main questions in cooperative game models are as follows:

1. What coalitions are formed?
2. If the whole coalition is formed, how do players share profit or cost?

In general, ideas such as the Shapley value, nucleolus, marginal share, Johnston index, Deegan-Packel index, and Shapley-Shubik index are proposed to share the benefits of cooperation. Each of these ideas has proposed a fair division of profits from cooperation on the basis of their assumptions (Abdoli, 2014).

3.1 Shapley value

For an N-person game with cooperation, Shapley has calculated the mean recipient of each player from coalition. This means that the amount received, $\boldsymbol{u_i^*}$**,** by player i is determined using this concept.

If player i joins a coalition C, his marginal productivity in the coalition is defined as equation 1:

$$\{V(C) - V(C - (i))\} \quad \textbf{(1)}$$

Assuming that coalition is generally formed from the coalition of one to two people ... and to N people and any sort of joining the coalition is possible, then $\boldsymbol{u_i^*}$ of player i shows the average marginal productivity in the game, which is defined by equation (2):

$$u_i^* = \sum_{\begin{cases} C \subset N \\ i \in C \end{cases}} \frac{(k-1)!(N-k)!}{N!} \{V(C) - V\big(C - (i)\big)\} \quad \textbf{(2)}$$

Where:

N: is total number of players

C: K is total number of players in coalition

$\frac{(k-1)!(N-k)!}{N!}$: is the possibility of the occurrence of each coalition

Equation (2) is obtained from all calculations per possible coalition of participant i.

Eq. (2) meets the following three principles:

A. Symmetry

If permutations of decision-makers i and j for a characteristic function of a decision-making process are equal to each other, then their payment efficiency after group decision-making must be the same. That means that for every permutation of r and N, equation (3) is established:

$$\begin{cases} V\big(r(C)\big) = V(C) \;\; ; \;\; \forall C \,\epsilon T \\ \qquad \longrightarrow \; u_{r(i)}^* = u_i^* \end{cases} \quad \textbf{(3)}$$

Received 23/04/2018
Approved 10/06/2018

B. Effectiveness

Effectiveness is shown in equation (4).

$$\sum_{i\in N} \boldsymbol{u}_i^* = \boldsymbol{V}(\boldsymbol{N}) \quad \textbf{(4)}$$

C. Assembly Law

Two desired and characteristic functions of $\boldsymbol{v}$ and $\boldsymbol{w}$ are considered, then equation (5) is as follows.

$$\boldsymbol{u}_i^*(\boldsymbol{v}+\boldsymbol{w}) = \boldsymbol{u}_i^*(\boldsymbol{v}) + \boldsymbol{u}_i^*(\boldsymbol{w})$$

(5)

Received value of player i in two separate games ***(v,w)*** is equal to his received value in a compound game that is consisted of those two separate games.

Based on the principles A to C, Shapley theorem is presented as follows.

Shapley theorem

Vector $\boldsymbol{u}_i^*$ exists and is essentially unique when it meets the above mentioned principles and its elements are obtained from Eq. (2). Shapley value $\boldsymbol{u}_i^*$ can characterize the power of player i in his sensitivity and effectiveness to win a coalition. The power will depend on the increase of productivity of a coalition after the player i has joined the coalition (Guillermo, 2015).

4. Research methodology

The present study is followed by the development of applied knowledge in a particular area. It is intended to use the findings to solve competitive location problems of bank branches and tries to answer a practical problem existing in the real world. Therefore, it is applied in terms of its objectives and is descriptive-survey in terms of research methodology. Research population also include all managers, experts and scholars in the field of banking at Sina Bank, who had the following features:

1) Having an experience of at least ten years in the banking industry

2) Having an organizational position of at least branch management and higher

The exact number of population of the study is unknown. To collect the required data, expert interviews and questionnaires have been used.

Step One: Determining the required number of branches in the region

Based on the existing standards, the number of required bank branches in the region is determined. It should be noted that based on the current situation, the number of bank branches per 100 thousand population is 27 bank branches, which is of course much higher than international standards.

Received 23/04/2018
Approved 10/06/2018

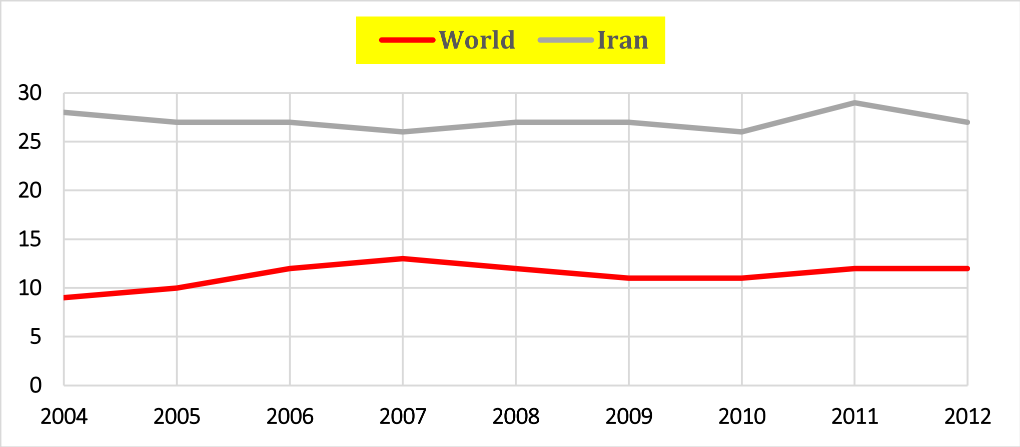

Figure 2: graph of the number of bank branches per 100 thousand population (Source: http://www.mckinsey.com/)

According to the above mentioned population and standard, the number of required branches is calculated. Therefore, output of this stage is shared with the sample and the raw number is summed up or reduced from the specified number of required branches in the region.

Step Two: Identifying the important criteria of selecting competitive location of banks

A list of important criteria in the choice of competitive location of facilities, specifically banks, is prepared and then, a number of the most important ones in the banking industry and financial sector are also chosen through interviews with the sample.

Step Three: Identifying candidate points

For each of the criteria identified in the second step, points candidate to create a new branch are proposed based on the number of required branches obtained in the first step.

Step Four: Creating the structure of cooperative game

Game theory is used to choose the best points based on identified competitive criteria. The game proposed in this section is such that each competitive criteria identified in the second step is considered as a player, which means we will have a number of players the same as the number of competitive criteria. In this game, each player (competitive criteria) will have game strategies to the number of candidate points in that criteria. For example, if 5 points are candidate in each criteria, each player has 5 game strategies. The purpose of this cooperative game (between competitive criteria) is to choose the best strategy (best candidate points) such that the marginal utility is maximized. Therefore, the structure of cooperative game is created up to this step.

Step Five: Calculating the utility of single coalition

In this step, the expert panel is asked to consider all candidate points in each criteria and give a score of 0 to 5 as the utility of candidate points in the criteria. For example, if there are four

Received 23/04/2018
Approved 10/06/2018

Revista Publicando, 5 No 15. (2). 2018, 1391-1419. ISSN 1390-9304

criteria (players) and 5 candidate points (game strategies) for each criterion, there will be a total of 20 candidate points and the expert panel gives a score of 0 to 5 to 20 candidate points for each criterion. For example, the candidate point 10 gets a utility score of 2 in criterion 3.

Step Six: Calculating the utility of dual coalition

In this step, all dual coalitions that can be formed among players are created and utility of these coalitions is taken into account. For example, if player 1 (competitive criteria 1) and player 2 (competitive criteria 2) form a coalition, they will have a utility. In order to calculate the utility of coalitions, sum of the utility of coalition members is used.

$$\boldsymbol{U_{F,C} = U_F(C) + U_C(C) = 4 + 2 = 6} \quad \textbf{(6)}$$

Step Seven: Calculating the utility of triple and multiple coalition

In this step, as step six, all triple coalitions are formed and utility of each coalition is obtained. All formable coalitions such as foursome and quintet coalitions are formed in this step and their utility is calculate like the previous step. Therefore, at the end of this step, utility of single, double, triple and ... coalitions is obtained.

Step Eight: Writing calculated utilities as normal tables

In this step, the results of coalitions' utility calculation done in steps five to seven are written in the form of normal tables common in game theory (2-dimentional tables) so that Nash equilibrium for each coalition can be extracted from these tables.

Step Nine: Calculating the value of characteristic function $\boldsymbol{V(C)}$ for each coalition

According to von Neumann and Morgenstern, the value of a characteristic function $\boldsymbol{V(C)}$ of a coalition in the normal form (two-dimensional) has features of a saddle point. As it is known, min-max of a row and min-max of a column coincide in a saddle point, this is because the allies in the row try to maximize their minimum utility and non-allies in the column try to minimize the maximum loss. Although non-allies may nevertheless have positive returns, but less return for allies could result in more return for them.

In general, the characteristics of a saddle point for a game in the normal form (2-dimentional) can be obtained by solving L-P linear programming (for each ally in the row).

Received 23/04/2018
Approved 10/06/2018

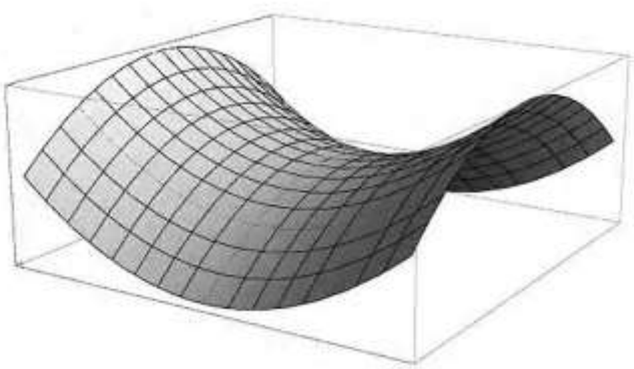

Figure 3: a saddle point (Source: Barron, E.N., (2013). Game theory: an Introduction. (2nd ed). Newjersey: John Wiley & Sons, Inc)

The $\boldsymbol{V(C)}$ defined for a two-dimensional and non-zero game (consisting of N players) observes $\boldsymbol{V(\phi) = 0}$ feature and covers more than additive. Therefore, if C_{ij} represents the utility of coalition of row i and column j of the coalition matrix (Coalition C) and P_i determines the choice probability (of combination of the strategies contained in the i-th row) for allies, so that the i-th row represents a combination of coalition's strategies and j-th column represents a combination of non-allies' strategies, then $\boldsymbol{V(C)}$ is obtained by solving (L-P) linear programming in equation (7):

$$\boldsymbol{V(C) = max: z}$$
$$\boldsymbol{st:}$$
$$\boldsymbol{z \leq \sum_i P_i * C_i \longrightarrow \forall j} \quad \textbf{(7)}$$
$$\boldsymbol{P_i \geq 0}$$
$$\boldsymbol{z: free}$$

Now to get the characteristic function of different coalitions of players (competitive criteria), Linear Programming (7) is written based on its coalition matrix and the $\boldsymbol{V(C)}$ value of any coalition is obtained by solving the model using Lingo software.

Step Ten: Calculating the Shapley value for each player

For an N-person cooperative game, the Shapley value of mean recipient of each player from coalition is calculated. This means that the $\boldsymbol{u_i^*}$ amount received by player i is determined with this concept.

If player i joins coalition C, his marginal productivity in the coalition is defined as equation (8):

$$\boldsymbol{\{V(C) - V(C - (i))\}} \quad \textbf{(8)}$$

Received 23/04/2018
Approved 10/06/2018

Assuming that coalition is generally formed from the coalition of one to two people ... and to N people and any sort of joining the coalition is possible, then $\boldsymbol{u_i^*}$ of player i shows the average marginal productivity in the game, which is defined by equation (9):

$$\boldsymbol{u_i^*} = \sum_{\left\{\substack{C\subset N \\ i\in C}\right.} \frac{(\boldsymbol{k}-\mathbf{1})!(\boldsymbol{N}-\boldsymbol{k})!}{\boldsymbol{N}!}\{\boldsymbol{V}(\boldsymbol{C}) - \boldsymbol{V}(\boldsymbol{C}-(\boldsymbol{i}))\} \qquad \textbf{(9)}$$

Where:

N: is total number of players

C: K is total number of players in coalition

$\frac{(\boldsymbol{k}-\mathbf{1})!(\boldsymbol{N}-\boldsymbol{k})!}{\boldsymbol{N}!}$: is the possibility of the occurrence of each coalition

Equation (9) is obtained from all calculations per possible coalition of participant i.

Shapley value $\boldsymbol{u_i^*}$ can characterize the power of player i in his sensitivity and effectiveness to win a coalition. The power will depend on the increase of productivity of a coalition after the player i has joined the coalition.

Using the value of characteristic function of $\boldsymbol{V}(\boldsymbol{C})$ calculated in step nine and the above explanation, Shapley value is calculated for each player (competitive criterion).

Step Eleven: choosing the optimal points of the candidate points

According to Shapley value obtained for players (competitive criteria), the order of power and their sensitivity in coalitions is determined.

5. Case study

To implement the framework provided in the study, region 6 of Tehran, Iran, is taken into account. It has a population of 269,034 people and an area of 2144 hectares that is limited to Hemmat highway in the north, Enqelab-Azadi axis in the south, Modarres Highway in the East, and Chamran Highway in the west. The area under the current circumstances and in accordance with approved official borders and boundaries is separated into seven districts and nineteen neighborhoods and is one of the most populated and most commercial areas of Tehran. Figure 5 shows the spatial location of this region.

Received 23/04/2018
Approved 10/06/2018

Figure 4: the spatial location of the sixth region of Tehran

Step One: Determining the required number of branches in the region

In the first step, the number of required bank branches in the region is determined. Table 1 shows the number of bank branches existing in this region.

Table 1: the number of bank branches existing in the sixth region of Tehran (source: researchers' findings)

Bank Name	Tejarat	Mellat	Melli	Saderat	Sepah	Keshavarzi	Refah	Maskan	Parsian	Post bank	D.E.B	N.E.B	Sina	Pasargad
Number of bank branches	73	68	61	51	41	23	22	13	10	3	6	6	5	5

The following steps are taken to determine the required number of branches:

Step 1: In this stage, based on the existing standards, the number of required bank branches in the region is determined. It should be noted that based on the current situation, the number of bank branches per 100 thousand population is 27 bank branches, which is of course much higher than international standards.

Due to the region's population that is 269000 people, the number of bank branches that must be placed in region 6 is equal to 73 bank branches. However, according to data in table (1), there are about 387 bank branches in this region, this means that there are 144 bank branches per 100,000 population. However, this is not unexpected because of the usage of region 6. The output of this step is a raw number of required number of branches in the region.

The results of step 1 shows that there is no need to create a new branch of Sina bank in region 6.

Received 23/04/2018
Approved 10/06/2018

Step 2: The raw number calculated in previous step is shared with the sample and given the current situation such as the expected need for
Sina bank in the region, managers' idea regarding the establishment of new branch and all points mentioned by the experts must be taken into account and then the raw number is summed up or reduced and the specified number of required branches in the region is obtained.
The expert panel of the study noted that there is definitely a need to develop bank branches in region 6 and argued that region 6, in fact, covers a large part of Tehran and it is even an important part of the country and the need to consider the development of branches is essential; as rival banks also have a strong focus on this region and statistics in Table 1 also shows. The expert panel believes that the excessive expansion of bank branches in an area is not suitable; therefore, they suggest that the development of branches should not be done without the necessary backings such as implementation of customer attraction systems. Finally, the decision maker team suggests that three new bank branched should be located in region 6 of Tehran.

Step Two: Identifying competitive location criteria

Table 2: competitive location criteria (Source: Miliotis et al. 2002; Farahani et al. 2010; Ashtiani, 2016)

Row	Criteria
1	Economic interests
2	Competitive strategy with other banks
3	Dense coverage
4	Existing infrastructure

Step Three: Identifying candidate points

In this step, for each of the criteria identified in the second step, points candidate to create a new branch are proposed based on the number of required branches obtained in the first step. Therefore, 3 points for each criterion and a total of 12 points are candidate points. Points that are candidate for each criterion are the best locations in terms of the criteria. In other words, candidate points in each criterion have sufficient utility in the criteria. This step is also conducted with the help of the expert team. Table 3 shows the candidate points in each criterion.

Table 3: identifying candidate points for each criterion

Criterion	Candidate points	Symbol
Economic interests	Valiasr – on the corner of Tarabande	F1

Received 23/04/2018
Approved 10/06/2018

	Iranshenasi Street	F2
	Farhang square	F3
Competitive strategy with other banks	Yadmane Sabz	C1
	Salmas square	C2
	Fatemi street – on the corner of Sanee	C3
Dense coverage	Vesal Street – on the corner of Felestin	I1
	Taleghani and Nejatollahi intersection	I2
	Somaye Street	I3
Existing infrastructure	Gharani – Apadana hospital	L1
	Ghaem Magham – Erfan	L2
	Alvand – Ahoora Mazda	L3

Step Four: Creating the structure of cooperative game

Game theory is used to choose the best points based on identified competitive criteria. The game proposed in this section is such that each competitive criteria identified in the second step is considered as a player, which means we will have a number of players the same as the number of competitive criteria. In this game, each player (competitive criteria) will have game strategies to the number of candidate points in that criteria. The purpose of this cooperative game (between competitive criteria) is to choose the best strategy (best candidate points) such that the marginal utility is maximized. Table 4 shows the created structure.

Received 23/04/2018
Approved 10/06/2018

Table 4: structure of the combination of different criteria strategies in a cooperative game with three candidate points for each criterion

structure of cooperative game										
	Economic interests									
		F1			F2			F3		
Dense coverage	I1	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3
		C2 L1	C2 L2	C2 L3	C2 L1	C2 L2	C2 L3	C2 L1	C2 L2	C2 L3
		C3 L1	C3 L2	C3 L3	C3 L1	C3 L2	C3 L3	C3 L1	C3 L2	C3 L3
	I2	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3
		C2 L1	C2 L2	C2 L3	C2 L1	C2 L2	C2 L3	C2 L1	C2 L2	C2 L3
		C3 L1	C3 L2	C3 L3	C3 L1	C3 L2	C3 L3	C3 L1	C3 L2	C3 L3
	I3	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3
		C2 L1	C2 L2	C2 L3	C2 L1	C2 L2	C2 L3	C2 L1	C2 L2	C2 L3
		C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3	C1 L1	C1 L2	C1 L3

Step Five: Calculating the utility of single coalition

Table 5: players' utility from strategies' combination

F_i	C_i	I_i	L_i	U_F	U_C	U_I	U_L
1	1	1	1	3	5	1	4
1	1	1	2	2	1	3	5
1	1	1	3	0	4	3	4

Received 23/04/2018
Approved 10/06/2018

1	1	2	1	4	3	0	1
1	1	2	2	4	2	1	1
1	1	2	3	0	4	1	0
1	1	3	1	3	2	1	2
1	1	3	2	2	1	1	4
1	1	3	3	2	3	0	3
1	2	1	1	4	4	2	0
1	2	1	2	3	3	3	1
1	2	1	3	2	4	3	3
1	2	2	1	1	2	4	4
1	2	2	2	2	0	1	0
1	2	2	3	3	3	0	1
1	2	3	1	0	3	5	4
1	2	3	2	5	1	0	3
1	2	3	3	3	3	3	0
1	3	1	1	3	3	0	3
1	3	1	2	0	4	2	5
1	3	1	3	2	1	5	2
1	3	2	1	4	1	3	2
1	3	2	2	1	4	2	4
1	3	2	3	5	5	5	3
1	3	3	1	2	0	3	1
1	3	3	2	1	4	5	3
1	3	3	3	3	2	2	1
2	1	1	1	1	4	1	2
2	1	1	2	1	2	3	3
2	1	1	3	3	1	4	2
2	1	2	1	1	4	5	3
2	1	2	2	2	3	1	4
2	1	2	3	5	0	2	1
2	1	3	1	1	1	1	0
2	1	3	2	4	4	5	1

Received 23/04/2018
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2	1	3	3	1	1	5	2
2	2	1	1	4	1	1	4
2	2	1	2	5	2	1	4
2	2	1	3	1	4	1	4
2	2	2	1	1	2	1	0
2	2	2	2	4	2	3	5
2	2	2	3	3	2	0	3
2	2	3	1	2	3	4	0
2	2	3	2	5	5	2	3
2	2	3	3	4	3	4	3
2	3	1	1	2	4	5	3
2	3	1	2	2	4	3	1
2	3	1	3	2	4	1	4
2	3	2	1	1	3	5	3
2	3	2	2	2	1	3	2
2	3	2	3	4	5	5	4
2	3	3	1	2	3	4	5
2	3	3	2	3	5	1	0
2	3	3	3	1	4	3	2
3	1	1	1	5	4	2	4
3	1	1	2	1	4	5	2
3	1	1	3	3	5	2	0
3	1	2	1	1	3	3	5
3	1	2	2	4	2	5	3
3	1	2	3	2	3	1	3
3	1	3	1	3	1	5	5
3	1	3	2	3	4	2	4
3	1	3	3	3	2	4	4
3	2	1	1	4	2	4	1
3	2	1	2	3	3	4	1
3	2	1	3	3	5	2	1

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3	2	2	1	2	2	3	4
3	2	2	2	3	1	3	2
3	2	2	3	3	4	5	3
3	2	3	1	2	3	2	2
3	2	3	2	3	5	3	4
3	2	3	3	5	3	4	4
3	3	1	1	3	1	4	2
3	3	1	2	3	4	2	5
3	3	1	3	1	1	5	4
3	3	2	1	4	5	2	2
3	3	2	2	0	3	1	1
3	3	2	3	3	5	3	4
3	3	3	1	5	3	5	2
3	3	3	2	4	5	3	1
3	3	3	3	1	5	3	1

Step Six and Seven: Calculating the utility of dual, triple and multiple coalition

Table 6: utility of coalition among players from strategies' combination

F_i	C_i	I_i	L_i	$U_{F,C}$	$U_{F,I}$	$U_{F,L}$	$U_{C,I}$	$U_{C,L}$	$U_{L,I}$	$U_{F,C,I}$	$U_{F,C,L}$	$U_{C,L,I}$	$U_{F,I,L}$	$U_{F,C,I,L}$
1	1	1	1	8	4	7	6	9	5	9	12	10	8	13
1	1	1	2	3	5	7	4	6	8	6	8	9	10	11
1	1	1	3	4	3	4	7	8	7	7	8	11	7	11
1	1	2	1	7	4	5	3	4	1	7	8	4	5	8
1	1	2	2	6	5	5	3	3	2	7	7	4	6	8
1	1	2	3	4	1	0	5	4	1	5	4	5	1	5
1	1	3	1	5	4	5	3	4	3	6	7	5	6	8
1	1	3	2	3	3	6	2	5	5	4	7	6	7	8
1	1	3	3	5	2	5	3	6	3	5	8	6	5	8
1	2	1	1	8	6	4	6	4	2	10	8	6	6	10
1	2	1	2	6	6	4	6	4	4	9	7	7	7	10
1	2	1	3	6	5	5	7	7	6	9	9	10	8	12

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1	2	2	1	3	5	5	6	6	8	7	7	10	9	11
1	2	2	2	2	3	2	1	0	1	3	2	1	3	3
1	2	2	3	6	3	4	3	4	1	6	7	4	4	7
1	2	3	1	3	5	4	8	7	9	8	7	12	9	12
1	2	3	2	6	5	8	1	4	3	6	9	4	8	9
1	2	3	3	6	6	3	6	3	3	9	6	6	6	9
1	3	1	1	6	3	6	3	6	3	6	9	6	6	9
1	3	1	2	4	2	5	6	9	7	6	9	11	7	11
1	3	1	3	3	7	4	6	3	7	8	5	8	9	10
1	3	2	1	5	7	6	4	3	5	8	7	6	9	10
1	3	2	2	5	3	5	6	8	6	7	9	10	7	11
1	3	2	3	10	10	8	10	8	8	15	13	13	13	18
1	3	3	1	2	5	3	3	1	4	5	3	4	6	6
1	3	3	2	5	6	4	9	7	8	10	8	12	9	13
1	3	3	3	5	5	4	4	3	3	7	6	5	6	8
2	1	1	1	5	2	3	5	6	3	6	7	7	4	8
2	1	1	2	3	4	4	5	5	6	6	6	8	7	9
2	1	1	3	4	7	5	5	3	6	8	6	7	9	10
2	1	2	1	5	6	4	9	7	8	10	8	12	9	13
2	1	2	2	5	3	6	4	7	5	6	9	8	7	10
2	1	2	3	5	7	6	2	1	3	7	6	3	8	8
2	1	3	1	2	2	1	2	1	1	3	2	2	2	3
2	1	3	2	8	9	5	9	5	6	13	9	10	10	14
2	1	3	3	2	6	3	6	3	7	7	4	8	8	9
2	2	1	1	5	5	8	2	5	5	6	9	6	9	10
2	2	1	2	7	6	9	3	6	5	8	11	7	10	12
2	2	1	3	5	2	5	5	8	5	6	9	9	6	10
2	2	2	1	3	2	1	3	2	1	4	3	3	2	4
2	2	2	2	6	7	9	5	7	8	9	11	10	12	14
2	2	2	3	5	3	6	2	5	3	5	8	5	6	8
2	2	3	1	5	6	2	7	3	4	9	5	7	6	9
2	2	3	2	10	7	8	7	8	5	12	13	10	10	15

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2	**2**	**3**	**3**	**7**	**8**	**7**	**7**	**6**	**7**	**11**	**10**	**10**	**11**	**14**
2	**3**	**1**	**1**	**6**	**7**	**5**	**9**	**7**	**8**	**11**	**9**	**12**	**10**	**14**
2	**3**	**1**	**2**	**6**	**5**	**3**	**7**	**5**	**4**	**9**	**7**	**8**	**6**	**10**
2	**3**	**1**	**3**	**6**	**3**	**6**	**5**	**8**	**5**	**7**	**10**	**9**	**7**	**11**
2	**3**	**2**	**1**	**4**	**6**	**4**	**8**	**6**	**8**	**9**	**7**	**11**	**9**	**12**
2	**3**	**2**	**2**	**3**	**5**	**4**	**4**	**3**	**5**	**6**	**5**	**6**	**7**	**8**
2	**3**	**2**	**3**	**9**	**9**	**9**	**10**	**10**	**10**	**14**	**14**	**15**	**14**	**19**
2	**3**	**3**	**1**	**5**	**6**	**7**	**7**	**8**	**9**	**9**	**10**	**12**	**11**	**14**
2	**3**	**3**	**2**	**8**	**4**	**3**	**6**	**5**	**1**	**9**	**8**	**6**	**4**	**9**
2	**3**	**3**	**3**	**5**	**4**	**3**	**7**	**6**	**5**	**8**	**7**	**9**	**6**	**10**
3	**1**	**1**	**1**	**9**	**7**	**9**	**6**	**8**	**6**	**11**	**13**	**10**	**11**	**15**
3	**1**	**1**	**2**	**5**	**6**	**3**	**9**	**6**	**7**	**10**	**7**	**11**	**8**	**12**
3	**1**	**1**	**3**	**8**	**5**	**3**	**7**	**5**	**2**	**10**	**8**	**7**	**5**	**10**
3	**1**	**2**	**1**	**4**	**4**	**6**	**6**	**8**	**8**	**7**	**9**	**11**	**9**	**12**
3	**1**	**2**	**2**	**6**	**9**	**7**	**7**	**5**	**8**	**11**	**9**	**10**	**12**	**14**
3	**1**	**2**	**3**	**5**	**3**	**5**	**4**	**6**	**4**	**6**	**8**	**7**	**6**	**9**
3	**1**	**3**	**1**	**4**	**8**	**8**	**6**	**6**	**10**	**9**	**9**	**11**	**13**	**14**
3	**1**	**3**	**2**	**7**	**5**	**7**	**6**	**8**	**6**	**9**	**11**	**10**	**9**	**13**
3	**1**	**3**	**3**	**5**	**7**	**7**	**6**	**6**	**8**	**9**	**9**	**10**	**11**	**13**
3	**2**	**1**	**1**	**6**	**8**	**5**	**6**	**3**	**5**	**10**	**7**	**7**	**9**	**11**
3	**2**	**1**	**2**	**6**	**7**	**4**	**7**	**4**	**5**	**10**	**7**	**8**	**8**	**11**
3	**2**	**1**	**3**	**8**	**5**	**4**	**7**	**6**	**3**	**10**	**9**	**8**	**6**	**11**
3	**2**	**2**	**1**	**4**	**5**	**6**	**5**	**6**	**7**	**7**	**8**	**9**	**9**	**11**
3	**2**	**2**	**2**	**4**	**6**	**5**	**4**	**3**	**5**	**7**	**6**	**6**	**8**	**9**
3	**2**	**2**	**3**	**7**	**8**	**6**	**9**	**7**	**8**	**12**	**10**	**12**	**11**	**15**
3	**2**	**3**	**1**	**5**	**4**	**4**	**5**	**5**	**4**	**7**	**7**	**7**	**6**	**9**
3	**2**	**3**	**2**	**8**	**6**	**7**	**8**	**9**	**7**	**11**	**12**	**12**	**10**	**15**
3	**2**	**3**	**3**	**8**	**9**	**9**	**7**	**7**	**8**	**12**	**12**	**11**	**13**	**16**
3	**3**	**1**	**1**	**4**	**7**	**5**	**5**	**3**	**6**	**8**	**6**	**7**	**9**	**10**
3	**3**	**1**	**2**	**7**	**5**	**8**	**6**	**9**	**7**	**9**	**12**	**11**	**10**	**14**
3	**3**	**1**	**3**	**2**	**6**	**5**	**6**	**5**	**9**	**7**	**6**	**10**	**10**	**11**
3	**3**	**2**	**1**	**9**	**6**	**6**	**7**	**7**	**4**	**11**	**11**	**9**	**8**	**13**

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3	3	2	2	3	1	1	4	4	2	4	4	5	2	5
3	3	2	3	8	6	7	8	9	7	11	12	12	10	15
3	3	3	1	8	10	7	8	5	7	13	10	10	12	15
3	3	3	2	9	7	5	8	6	4	12	10	9	8	13
3	3	3	3	6	4	2	8	6	4	9	7	9	5	10

Step Eight: Writing calculated utilities as normal tables

In this step, the results of coalitions' utility calculation done in steps five to seven are written in the form of normal tables common in game theory (2-dimentional tables) so that Nash equilibrium for each coalition can be extracted from these tables. To avoid the presentation of multiple tables obtained from coalitions, the normal form of utility of coalitions F and C is explained using tables (5) and (6).

Table 7: normal form of utility of coalition (F, C)

	Ii	1	1	1	2	2	2	3	3	3
	Li	1	2	3	1	2	3	1	2	3
F_i	C_i									
1	1	8	3	4	7	6	4	5	3	5
1	2	8	6	6	3	2	6	3	6	6
1	3	6	4	3	5	5	10	2	5	5
2	1	5	3	4	5	5	5	2	8	2
2	2	5	7	5	3	6	5	5	10	7
2	3	6	6	6	4	3	9	5	8	5
3	1	9	5	8	4	6	5	4	7	5
3	2	6	6	8	4	4	7	5	8	8
3	3	4	7	2	9	3	8	8	9	6

Step Nine: Calculating the value of characteristic function $\mathbf{V(C)}$ for each coalition

The $\mathbf{V(C)}$ defined for a two-dimensional and non-zero game (consisting of N players) observes $\boldsymbol{V(\phi) = 0}$ feature and covers more than additive. Then, $\mathbf{V(C)}$ is obtained by solving (L-P) linear programming in equation (7).

Table 8: characteristic function of coalition

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V(F)	V(C)	V(I)	V(L)	V(F,C)	V(F,I)	V(F,L)	V(I,L)	V(C,L)	V(C,I)	V(F,C,I)	V(F,C,L)
1.25	1.818182	1	0.9	5.310345	5.57143	5.44118	5.251917	5.516854	5.5	10.8636	11.18182

Step Ten: Calculating the Shapley value for each player

Shapley value u_i^* can characterize the power of player i in his sensitivity and effectiveness to win a coalition. The power will depend on the increase of productivity of a coalition after the player i has joined the coalition. Using the value of characteristic function $\mathbf{V}(\mathbf{C})$ calculated in the previous step and equations 8 and 9, Shapley value is calculated for each player (competitive criterion).

Table 9: Shapley value of players

	shapley value
F	4.3966655
C	5.377244
I	4.586124667
L	4.639965333

Step Eleven: choosing the optimal points of the candidate points

According to Shapley value obtained for players (competitive criteria), the order of power and their sensitivity in coalitions is determined. It is such that any criterion with high Shapley value has the highest impact on the process of utility creation. Accordingly, the process to choose optimal points will be as follows:

According to Shapley value obtained for players, C> L> I> F is the order of power and their sensitivity in the coalition. Therefore, in choosing to run a combination of strategies, competitive strategy with other banks has a decisive role, then according to the category of competitive strategy with other banks, the criterion of existing infrastructure, dense covering criterion, and economic interests deal with choosing a strategy to get the best payment, respectively.

6. Discussion and conclusion

According to 81 different combinations of strategies and amounts received by each player, in 12 strategy combinations, the amount received by the criterion of competitive strategy with other banks is 5. Therefore, the criterion of competitive strategy with other banks will choose its strategy from among these 12 strategy combinations.

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Table (10) shows combinations in which the criterion of competitive strategy with other banks had 5 recipients. After the criterion competitive strategy with other banks, the criterion of existing infrastructure tries to maximize his recipients. Therefore, the criterion of existing infrastructure chooses a combination that gets the highest recipient. The process will continue until the last player so that a certain strategy combination is selected. The selection process is shown in figure (5).

If the criterion of competitive strategy with other banks chooses strategy 1 from among the combinations with 5 recipient, the criterion of existing infrastructure chooses strategy 1 from among the two strategies of 1 and 3 with 4 and 0 recipients, respectively. After the selection of the criterion of existing infrastructure, no other choice is left to the next players, i.e. the dense coverage and economic interests, and they have to choose strategies 1 and 1 with 1 and 3 recipients, respectively. Finally, the strategy combination of $C_1L_1I_1$ is chosen and there is no need for a fourth choice.

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Table 10: final combination of strategies to be chosen by players

C	L	I	F	C-u	l-u	i-u	F-u
3	3	2	1	5	3	5	5
3	3	2	2	5	4	5	4
3	2	3	2	5	0	1	3
3	1	2	3	5	2	2	4
3	3	2	3	5	4	3	3
3	2	3	3	5	1	3	4
3	3	3	3	5	1	3	1
2	2	3	2	5	3	2	5
2	3	1	3	5	1	2	3
2	2	3	3	5	4	3	3
1	1	1	1	5	4	1	3
1	3	1	3	5	0	2	3

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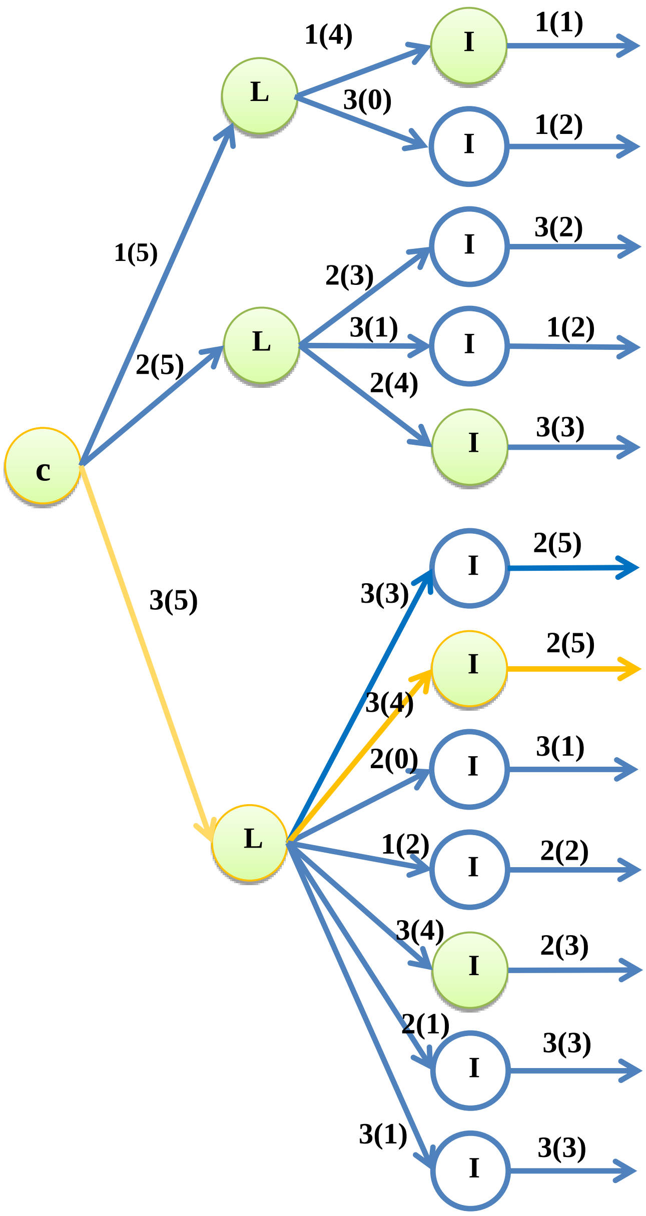

Figure 5: The process of choosing the best location

If the criterion of competitive strategy with other banks chooses strategy 2, the criterion of existing infrastructure chooses the combination of $C_2L_2I_3$ with 4 recipients, which is the highest one. After the selection of the criterion of existing infrastructure, no other choice is left to the next players and they accept the selected strategy combination.

If the criterion of competitive strategy with other banks chooses strategy 3, the criterion of existing infrastructure has two choices of $C_3L_3I_2$ and $C_3L_3I_2$, in which the highest recipients are the same. Therefore, the criterion of dense coverage has a choice and chooses $C_3L_3I_2$ from

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among the two existing combinations because of the highest recipient. The process of choosing the strategy is shown in figure (5).

According to this description, Fatemi Street – on the corner of Sanee, Alvand – Ahoora Mazda, and Taleghani and Nejatollahi intersection are chosen as optimal points to establish bank branches, which were the best points from the viewpoint of experts.

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Received 23/04/2018
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Received 23/04/2018
Approved 10/06/2018