Comparison of the Modern Earth's Gravity Field Models

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ABSTRACT

Investigations of structure of gravitational fields of Earth and other celestial bodies represent an important and complex problem which relevance is generated by increase of flight activity in Solar system and growth of requirements to the accuracy of the solution of the navigation and geophysical tasks connected with it. Article is devoted to comparison with each other of the models of gravitational field of Earth constructed according to the space missions of GRACE and GOCE. Special attention in work is paid to comparison of only satellite models with the models combined for which creation along with satellite data materials of gravimetric data and a satellite altimetry of a surface of the World Ocean data were used. The new technique of comparison of models by calculation of differences of values of couples of harmonious coefficients of these two models having identical indexes is offered. Results of calculations show the good consent of the compared models of gravitational potential of the Earth in low-frequency area.

Keywords: gravitational field of Earth, space GRACE, GOCE programs, comparison of models of a geopotential.

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1. INTRODUCTION

Determination of the figure and external gravitational field of Earth throughout many centuries is considered as one of the most fundamental problems of natural sciences. Today development of advanced space means and technologies opens new opportunities and prospects of its decision. Let's notice in this regard that the problem of a research of gravitational fields of Earth and other bodies of Solar system has complex character and represents a sphere of application of interests of various fields of science and technology as detailed and reliable information about structure of fields of forces of an attraction of celestial bodies is necessary for the solution of many scientific and applied tasks.

The successful implementation of the missions CHAMP (Flury J. 2006), GRACE (Sheard B. S. 2008), GOCE (Rummel R. 2005) dated by the beginning of our century opened new opportunities of a research of structure of gravitational field of Earth. Low-orbital satellites at the same time play a role of trial masses, change of the direction and speed vector module at the movement in which non-uniform field of force of an attraction bears information on features of structure of this field. Application of new methods and technologies of Satellite-to-Satellite Tracking (Shin-Chan Nan. 2006) and a Satellite Gravity Gradiometry (Freeden W. 2007) allowed not only to detail essentially our knowledge of structure of gravitational field of Earth, but also to obtain data on changes of this structure eventually. It is very important that in each of the considered projects a key role is played by high-precision positioning of low-orbital satellites by means of a binding to the constellation of satellites of the global GPS navigation system. The divergences of results of a GPS binding of the CHAMP, GRACE, GOCE satellites received in real time with data of a land laser location make 2-3 cm that testifies to efficiency of use of global navigation and geodetic satellite systems for high-precision positioning of low-orbital objects.

Thus, the global navigation satellite systems, no doubt, representing one of the most important technical achievements of the end of the last century, it is directly integrated not only in navigation and geodesy, but also in gravimetry, geophysics and other sciences about Earth.

Today various research teams and authors constructed tens of models of geopotential differing with sets of basic observation data, methods of their processing, the amounts

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of the estimated unknowns, and accuracy of the obtained results. The modern variety of such models staticizes the questions of their comparison, verification and accuracy considered by us in this article for the purpose of further use for the solution of various problems of geodesy, gravity measurements, celestial mechanics and geophysics.

2. METHODS OF MODELS COMPARISON FOR GRAVITATIONAL POTENTIAL

For Earth and planets of terrestrial group the most common form of the numerical and analytical description of the field of force of an attraction is decomposition of gravitational potential $V(\rho,\varphi,\lambda)$ in a row of spherical functions:

$$V(\rho,\varphi,\lambda)=\frac{GM}{\rho}\left[1+\sum_{n=2}^{N}\sum_{m=0}^{n}\left(\frac{R}{\rho}\right)^{n}(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda)P_{nm}(\sin\varphi)\right] \quad (1)$$

set of final number of dimensionless harmonious coefficients $\{C_{nm},S_{nm}\}$, $(n=2,3,\ldots N;\quad m=0,1,\ldots n)$ who (N -dimension of model), is represented by a set of numerical parameters of model of gravitational potential of the considered celestial body. Let's remind also that the gravitational potential of a body $V(\rho,\varphi,\lambda)$ is connected with a vector of the Newtonian force of an attraction $\vec{F}(\rho,\varphi,\lambda)$ the known ratio:

$$grad V(\rho,\varphi,\lambda)=\vec{F}(\rho,\varphi,\lambda).$$

In decomposition (1) symbols ρ,φ,λ designated spherical coordinates (planetocentric radius vector, latitude, longitude) of a space point in which value of potential $V(\rho,\varphi,\lambda)$,by a symbol $P_{nm}(\sin\varphi)$ - the attached Legendre's function of degrees *n*, and of order *m* is calculated.

It is with each other possible to carry out comparison of the models set by a set of numerical values of harmonic coefficients of decomposition of gravitational potential in a row of spherical functions by three complementary ways:

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- comparison of individual values of separate coefficients for everyone n - an index of degree and m - an index of an order;
- by means of various criteria calculated for each fixed degree on all coefficients of various orders of this degree;
- comparison of values which are in advance chosen any functional of potential, calculated on the compared models, obtained as a result of summation of all members of the corresponding series. In most cases, it is convenient to carry out this comparison in terms of change of heights of geoid.

Now the models of gravitational field of Earth set by set of harmonious coefficients can be divide into two classes: only satellite models and combined models.

The class of only satellite models possesses models which values of parameters are derived from only onboard satellite measurements. The maximum value of an index of degree of n of decomposition in a row (sometimes called by dimension of model) such models, as a rule, does not exceed the size n=360. It is clear, that only satellite models are intended, first of all, for obtaining navigation characteristics of objects at satellite heights and therefore are mainly used for creation of adequate models of the orbital motion of artificial satellite.

The class of the combined models is formed by a set of the models, constructed on set of diverse onboard and ground-based measurements including gravimetric measurements and data of a satellite altimetry of a surface of the World Ocean. The specified circumstance allows to increase considerably number of the estimated parameters of model of gravitational potential of Earth, increasing dimension of model to n=2000 and more.

For more detailed research on materials of the website ICGEM (International Center Global Earth Model (ICGEM) of URL: <http://icgem.gfz-potsdam.de/tom_longtime>. - Checked 21.08.2017.) we chose about 4 models of each class (tab. 1) constructed according to various observational data. At the same time both the only satellite, and combined models were compared with each other in each group (class) and between groups. Two (EGM2008 (Gilardoni M. 2008), EIGEN-6S3stat) from four combined models deserve the name multiple parameter as the number entering each of models of parameters - harmonic coefficients makes several millions.

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Table 1. The considered models of gravitational potential of the Earth

Model number	Model name according (Flury J. 20061)	Year	N	Types of data
Only satellite models				
(1S)	ITG-GRACE 2010S	2010	180	GRACE
(2S)	GOCO03S	2012	250	GRACE, GOCE
(3S)	GO_CONS_GCF_2_TIM_R5	2014	280	GOCE
(4S)	GGM05G	2015	240	GRACE, GOCE
Combined models				
(5C)	GGM03C	2009	360	GRACE, gravity, altimetry
(6C)	EIGEN -6C3stat	2014	1949	GOCE, GRACE,LAGEOS, gravity, altimetry
(7C)	EGM2008	2008	2190	GRACE, gravity, altimetry
(8C)	GGM05C	2016	360	GRACE, GOCE, gravity, altimetry

Important feature of the chosen models is use at their creation of GRACE and GOCE missions observations data marked by high precision and permission of details of structure of the field.

Among various techniques of comparison of models (Tsoulis D. 2010), (Novlyanskaya I. O.2011) for ensuring presentation of results when comparing separate coefficients of a row (1) we used the technique described in work (Kashcheev R. A.2008).

In geocentric spherical system of coordinates ρ, φ, λ of heights of a geoid are modeled by decomposition in a row of spherical functions:

$$N(\varphi,\lambda)=\sum_{n=2}^{\hat{n}}\sum_{m=0}^{n}\sum_{j=1}^{2}N_{nmj}(\varphi,\lambda), \qquad (2)$$

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$$N_{nm1}(\varphi,\lambda) = R \cdot \overline{C}_{nm} \cdot \cos m\lambda \cdot \overline{P}_{nm}(\sin\varphi),$$

$$N_{nm2}(\varphi,\lambda) = R \cdot \overline{S}_{nm} \cdot \sin m\lambda \cdot \overline{P}_{nm}(\sin\varphi),$$

where $\overline{C}_{nm}$, $\overline{S}_{nm}$ - full normalized harmonic coefficients of decomposition of gravitational potential of Earth in a series of spherical functions of degree n and of an order m which set of values makes concrete model of gravitational field of the Earth;

$\overline{P}_{nm}(\sin\varphi)$ - full normalized attached Legendre's functions of n order and m degrees;

R - mean radius of the Earth.

We will carry out comparison of two various models (1) and (2) of a geopotential as follows. Let's designate sets corresponding to the specified models of harmonic coefficients $\{\overline{C}^{(1)}_{nm}, \overline{S}^{(1)}_{nm}\}$ and $\{\overline{C}^{(2)}_{nm}, \overline{S}^{(2)}_{nm}\}$. We will write down spherical function N_{nmj} for each of models in a look:

$$N^{(1)}_{nmj}(\varphi,\lambda) = \left\{ \begin{matrix} \overline{C}^{(1)}_{nm} \cos m\lambda \\ \overline{S}^{(1)}_{nm} \sin m\lambda \end{matrix} \right\} \overline{P}_{nm}(\sin\varphi) \quad \begin{matrix} j=1 \\ j=2 \end{matrix},$$

$$N^{(2)}_{nmj}(\varphi,\lambda) = \left\{ \begin{matrix} \overline{C}^{(2)}_{nm} \cos m\lambda \\ \overline{S}^{(2)}_{nm} \sin m\lambda \end{matrix} \right\} \overline{P}_{nm}(\sin\varphi) \quad \begin{matrix} j=1 \\ j=2 \end{matrix}.$$

Then we form differences:

$$dN_{nmj}(\varphi,\lambda) = N^{(2)}_{nmj}(\varphi,\lambda) - N^{(1)}_{nmj}(\varphi,\lambda) =$$
$$= R \cdot \left\{ \begin{matrix} d\overline{C}_{nm} \cos m\lambda \\ d\overline{S}_{nm} \sin m\lambda \end{matrix} \right\} \overline{P}_{nm}(\sin\varphi) \quad \begin{matrix} j=1 \\ j=2 \end{matrix}, \qquad (3)$$

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where it is designated: $d\overline{C}_{nm} = \overline{C}^{(2)}_{nm} - \overline{C}^{(1)}_{nm}$, $d\overline{S}_{nm} = \overline{S}^{(2)}_{nm} - \overline{S}^{(1)}_{nm}$.

Expression (3) represents the elementary change of height of a geoid $dN(\varphi,\lambda)$, caused by change of value of coefficient $\overline{C}_{nm}$ (or $\overline{S}_{nm}$) upon transition from model (1) to model (2). Let's square equality (2), we will increase on $\frac{1}{4\pi}\cos\varphi d\varphi d\lambda$ and we will integrate on the sphere □ single radius:

$$\frac{1}{4\pi}\iint_{\omega}\left[d\sigma_{nmj}(\varphi,\lambda)\right]^2\cos\varphi d\varphi d\lambda =$$

$$= R^2 \left\{\begin{pmatrix} d\overline{C}_{nm} \\ d\overline{S}_{nm} \end{pmatrix}^2\right\}\frac{1}{4\pi}\iint_{\omega}\left[\overline{P}_{nm}(\sin\varphi)\begin{Bmatrix}\cos m\lambda \\ \sin m\lambda\end{Bmatrix}\right]^2\cos\varphi d\varphi d\lambda.$$

Owing to orthogonality of fully normalized elementary spherical functions on the single sphere, we have:

$$\frac{1}{4\pi}\iint_{\omega}\left[\overline{P}_{nm}(\sin\varphi)\begin{Bmatrix}\cos m\lambda \\ \sin m\lambda\end{Bmatrix}\right]^2\cos\varphi d\varphi d\lambda = 1,$$

from where in turn:

$$\left(\frac{1}{4\pi}\iint_{\omega}\left[dN_{nmj}(\varphi,\lambda)\right]^2 d\omega\right)^{\frac{1}{2}} = R\cdot\begin{Bmatrix} d\overline{C}_{nm} \\ d\overline{S}_{nm}\end{Bmatrix}\begin{matrix} j=1 \\ \\ j=2\end{matrix}. \quad (4)$$

The values equated each other by expression (4) have dimension of length and can be considered as a measure of a contribution to the change of excess of a planetary geoid in some point caused by change of coefficients $\overline{C}_{nm}$ and $\overline{S}_{nm}$ upon transition from model (1) to model (2). Let's notice that this measure has the global integrated character which is not allowing estimating influence of the considered

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harmonious coefficient on changes of a figure of a geoid in local area or in a concrete point of the land surface.

Thus, the offered technique corresponds to paired comparison by criterion (4) sets belonging to two compared models of couples of harmonic coefficients having identical values of indexes.

3. RESULTS OF COMPARISON OF MODELS OF A GEOPOTENTIAL

Analyzing results of comparison of four (1S, 2S, 3S, 4S according to table 1) only satellite models, we come to the following conclusions.

Only satellite models are very close to each other in low-frequency (approximately for n <100) area of decomposition of a geopotential describing global features of structure of gravitational field of Earth. In process of increase of values of an index of degree of n there is a mismatch of harmonic coefficients, at first sectorial, and then and tesseral.

The zonal component of parameters practically coincides at all compared models, except for model (3S) for which the mismatch of coefficients at the level of 1 mm arises, since n=50. The mismatch of sectorial coefficients reaches this level, since n=100, and tesseral with n=160 if one of the compared models is the model (4S) and with n=200 in other cases.

The models (3S) and (4S) showing a mismatch of parameters only in sectorial and zone areas are the closest to each other. For model (4S) the mismatch with other models of a number of the tesseral coefficients belonging to range 115 <n <135 is also observed.

When comparing with each other of coefficients of four combined models of gravitational potential of Earth (5C), (6C), (7C), (8C), the high consent of values of parameters in the field of low frequencies (n <100), except for a number of harmonic coefficients, sectorial and close to them, for which the mismatch at the level of more than 1 mm arises, since n=80 is also observed.

In the field of average frequencies (100 <n <250) the mismatch of numerical values of parameters of the majority of couples of compared combined models exceeds the threshold level of 1 mm that is most brightly shown in couples with participation of model (5C), and least - in couples with participation of model (8C); at the same time the model (5C) at average frequencies finds the smallest correlation with other three considered models. Upon transition to area of higher frequencies (n> 250) similarity of zonal and tesseral coefficients for couples of combined models increases, however, a

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mismatch of coefficients with close values of the first and second indexes again, that is for sectorial components, remains.

The consent of values of parameters of models in the field of low frequencies (n<100) is observed also in case of comparison of couples of coefficients of various only satellite and combined models. In the fields of averages (100<n<250) and high (n>250) frequencies the combined models (5C) and (7C) will be poorly correlated with all satellite models while values of parameters of the combined models (6C) and (8C) are much closer to the corresponding values of parameters of the only satellite models compared to them, except for a number of sectorial coefficients.

4. DISCUSSIONS

Thus, are characteristic proximity of values of parameters both the only satellite, and combined models in the field of low frequencies of all options of the choice of couples of models compared by us in this work (n <100) that allows to speak about high degree of adequacy of a low-frequency component of modern models of gravitational field of Earth. In the field of average and high frequencies the consent of only satellite and combined models weakens that, apparently, is a consequence of attraction of additional measuring data for creation of the combined potential models.

5. CONCLUSION

Models of gravitational field of Earth divide into 2 main groups: only satellite and combined which are in turn various structure of basic observation data, processing methods, the number of the estimated parameters and accuracy of results. In turn existence of constantly increasing quantity of models of a geopotential staticized questions of assessment of their accuracy and comparison. One of techniques of comparison of models is paired comparison of separately taken harmonic coefficients having identical values of indexes of degree of n and orders of m.

Comparison of only satellite models showed their almost full coincidence in the field of low frequencies which describes global features of a geopotential. At increase of values n of distinction of models are observed at first in sectorial, and then and in tesseral harmonicas.

Comparison of the combined models also shows their consent in low-frequency area and a small mismatch in the field of average and high frequencies.

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In the conclusion it should be noted almost full coincidence of parameters both only satellite, and combined models at values of degree of n <100. With increase of values n the consent of all considered models weakens that can be correlated with a combination of various data obtained from various sources.

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